Some important features of three-dimensional disturbances and the "roof shingle" shape proposed by Wortmann[1] have been reproduced by the present calculation method. This fact shows that the eigenfunctions obtained from linear theory can be used in some extent to describe three-dimensional disturbance structure.

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Contribution to the Reynolds Stress Model as Applied to Near-Wall Region

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I. Introduction

SOME complicated turbulent flowfields have recently been solved by using the Reynolds stress (RS) model. This indicates that the RS model has become a practical approach which can be used to satisfy technical demands. However, with the exception of Hanjalic and Launder,[1] there have been few authors who have solved the whole flowfield, including the near wall region. Here, it is emphasized that the near wall region used in this Note includes the solution up to the wall.

Since the RS model usually used[1-7] is based on the considerations of isotropic turbulent flow, it can not be applied directly to a strongly anisotropic flow, such as a flow near a wall. Hence, in most reports,[1-7] the near wall region is excluded from calculations. The wall boundary conditions are applied in the proximity of the wall and the flow properties are matched, for instance, to the law of the wall.

Hanjalic and Launder (HL) succeeded in solving the whole flowfield including the near wall region by estimating the direct viscous effects on various transport processes and introducing a function to correct the profile of the Reynolds shear stress near the wall. The HL model is simplified by using two experimental approximations and includes only the transport equation of the Reynolds shear stress, $\overline{uv}$.

For engineers, it is desirable to solve general three-dimensional flow by a complete RS model. There has been no attempt to solve the whole flowfield up to a wall with this approach.

This Note presents a model for solving the whole flowfield including the near wall region by using a complete RS model. The present method is based on the idea given in the next section, in addition to Hanjalic and Launder's considerations about the direct viscous effects. The newly developed model was applied to two-dimensional turbulent flows. The results showed good agreement with the experimental data in the whole flowfield including the near wall region.

II. Development of Model

The pressure-strain correlation term arising in the RS model is also called the redistribution term of turbulent energy and has a characteristic of making a flow isotropic. From this characteristic, if the function of this term is restrained in a near wall region, it is anticipated that the model can describe the anisotropic flow in that region without serious error.

Using this idea, functions multiplying the pressure-strain terms were introduced in the transport equations of Reynolds stress. The transport equations for two-dimensional shear flow are, then, written as follows.

$$\frac{D\overline{u_iu_j}}{Dt} = -\left(\overline{u_ju_k}\frac{\partial U_i}{\partial x_k} + \overline{u_iu_k}\frac{\partial U_j}{\partial x_k}\right) + \nu\frac{\partial^2 \overline{u_iu_j}}{\partial x_k^2} + \pi_{ij}F$$
$$+ C\frac{\partial}{\partial x_k}\left[\frac{k}{\epsilon}\left(\overline{u_iu_\ell}\frac{\partial\overline{u_ju_k}}{\partial x_\ell} + \overline{u_ju_\ell}\frac{\partial\overline{u_ku_i}}{\partial x_\ell} + \overline{u_ku_\ell}\frac{\partial\overline{u_iu_j}}{\partial x_\ell}\right)\right]$$
$$-2\left(\frac{1-f_s}{3}\delta_{ij} + \frac{\overline{u_iu_j}}{2}\frac{f_s}{k}\right)\epsilon$$

where π_{ij} is the pressure-strain correlation term which is the same one as Hanjalic and Launder used. F represents the introduced functions, which were determined by trial and error, and f_s is the function proposed by Hanjalic and Launder to deal with the viscous effects of the dissipation ϵ.

The following three-F functions were used in each transport equation of Reynolds stress.

For $\overline{uv}$:

$$F1 = \exp\{-0.4/[1+(R_t/50)^2]\}$$

For $\overline{u^2}, \overline{v^2}$:

$$F2 = \exp[-1.2/(1+R_t/200)]$$
$$= 0.31 - 0.12(5-R_t)^{0.5} \qquad R_t < 5$$

For $\overline{w^2}$:

$$F3 = \exp[-1.2/(1+R_t/50)]$$
$$= 0.336 - 0.13(5-R_t)^{0.5} \qquad R_t < 5$$

where R_t is the turbulent Reynolds number, $R_t = k^2/\nu\epsilon$.

Here, it must be emphasized that these functions are significant as a combination and must not be considered separately. These functions are one of the combinations able to produce reasonable results in the near wall region; other

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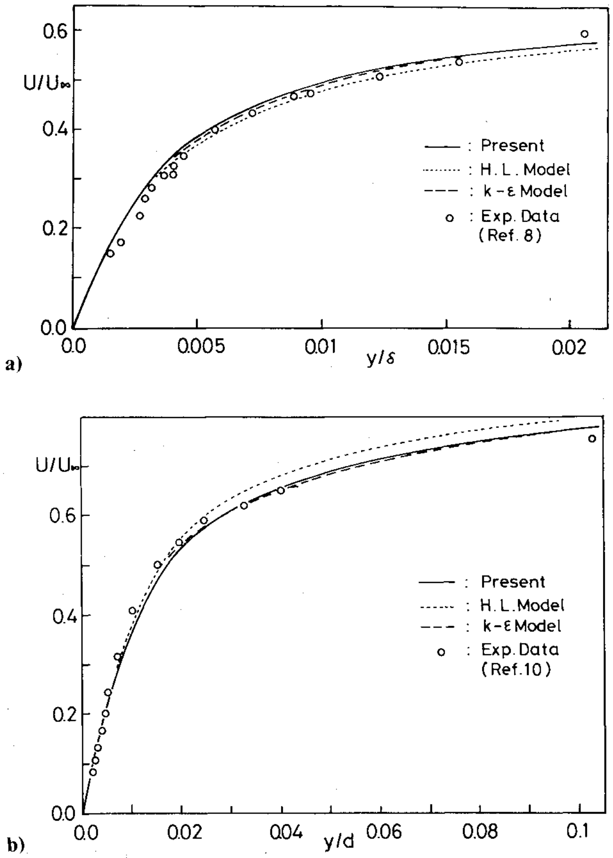

Fig. 1 Mean velocity profiles near wall region a) flat plate boundary layer, $Re\delta = 7.5 \times 10^4$ and b) channel flow, $Red = 30800$.

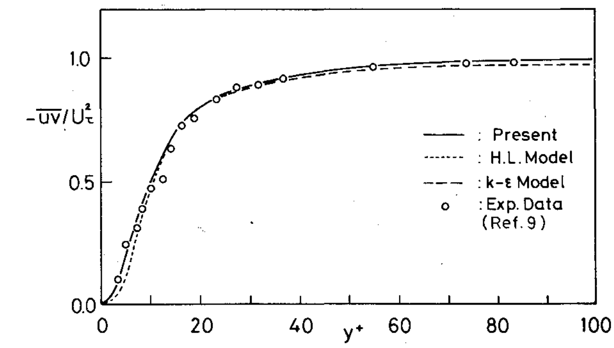

Fig. 2 Reynolds shear stress profile near wall region; flat plate boundary layer, $Re\delta = 7.5 \times 10^4$.

combinations of functions have not been investigated. Naturally, a large number of functions were tested to replace these three functions with a single one. However, no single function could produce good results in the wall region for all flow properties.

This suggests that the approximations and the modeling for each term arising in the transport equations are not the same order for Reynolds stress elements and in some cases are not suitable. This question is not discussed any further here.

Results and Discussion

The newly developed RS model was applied to two-dimensional cases: flat plate and channel flows. In Figs. 1-3, typical results in the near wall region are shown with those of the k-ϵ model and the HL model. The Reynolds stress elements of the HL model were calculated from the approximations: $uv/(u^2v^2)^{1/2} \cong 0.47$ and $3/4\ (u^2+v^2) \cong k$. The

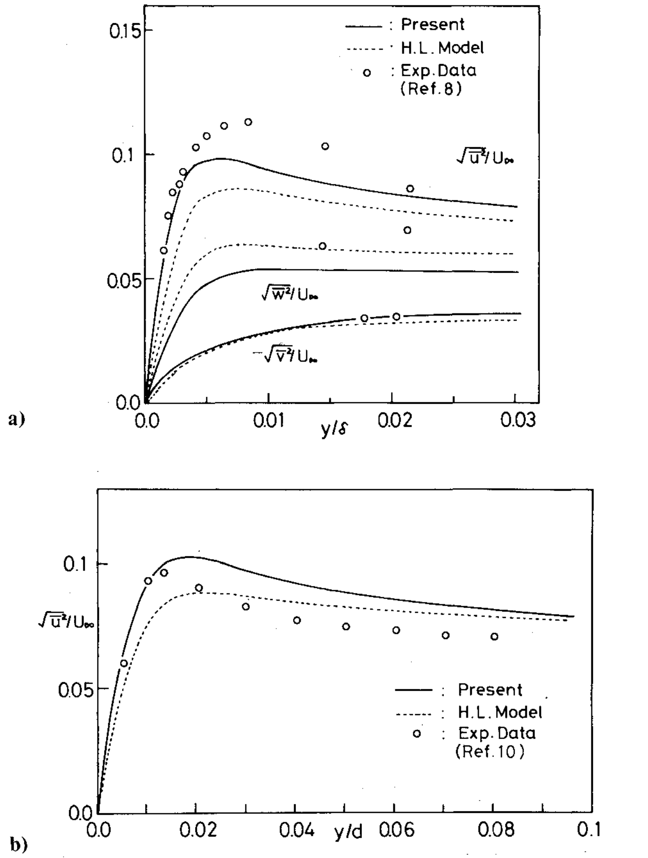

Fig. 3 Turbulence intensity profiles near wall region a) flat plate boundary layer, $Re\delta = 7.5 \times 10^4$ and b) channel flow, $Red = 30800$.

results obtained are reasonable in comparison with the experimental data and there is not any serious error. Agreement in the outer flowfield was as good as in the near wall region.

The applicability of the present model should be examined for various flowfields. This Note shows that the complete RS model, modified with the functions introduced in the preceding section can describe the whole flowfield including the near wall region without serious error, at least for two dimensional flows.

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